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'LETTERS' SECTION

On Relativistic Quantum Field Theory in Configuration Space

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Let us consider a relativistic quantum field theory. Let $a_{\mathbf{k}}^*$ be the creation operator of a free particle of momentum $\mathbf{k}$ and $\phi^{(-)}(\mathbf{x})$ be the corresponding standard creation part of the configuration space field at time equal to zero. Here the word 'standard' means that $\phi^{(-)}(\mathbf{x})$ is obtained from $a_{\mathbf{k}}^*$ by means of a Fourier transformation with the relativistic weighing factor (e.g. k_0^{-1}). A non-standard creation configuration field $\phi'^{(-)}(\mathbf{x})$ will be a different linear combination of the $a_{\mathbf{k}}^*$'s. It has been stated that while $a_{\mathbf{k}}^*|0\rangle$ is an eigenstate of momentum the state $\phi^{(-)}(\mathbf{x})|0\rangle$ is not an eigenstate of position (cf. e.g. Lurié, 1968). If right, this ill-configuration behaviour of the field (which does not happen in the non-relativistic case) would obscure the physical interpretation of the theory. In order to have a better configuration field (if it exists), one should try a field $\phi'^{(-)}(\mathbf{X})$ which would be a different linear combination of the $a_{\mathbf{k}}^*$'s, i.e. the creation part of a non-standard configuration field; cf. Wightman & Schweber (1955) who have shown that the change $\phi^{(-)}(\mathbf{x}) \to \phi'^{(-)}(\mathbf{X})$ corresponds to the replacement, in the one-particle subspace, of an incorrect position operator $\mathbf{x}_{\text{op}}$ by the correct one $\mathbf{X}_{\text{op}}$. It would then be expected that

$$X_{\text{op}}^i\,\phi'^{(-)}(\mathbf{X})|0\rangle = X^i\,\phi'^{(-)}(\mathbf{X})|0\rangle, \qquad i = 1, 2, 3\dagger \tag{1}$$

(Notice that $\phi'^{(-)}(\mathbf{X})|0\rangle$ may still have non-point-like behaviour, see Wightman & Schweber, 1955, equation (57).) At the time Wightman & Schweber (1955) wrote their article it seemed that the right position operator

† Since $\mathbf{X}$ belongs to the continuum, it should not be surprising that $\|\phi'^{(-)}(\mathbf{X})|0\rangle\|$ diverges. In order not to go out of the correct (finite norm) state vector space, $\phi(\mathbf{X})$ should be replaced by an average of ϕ using a well-behaved c-number weighing factor strongly localised at $\mathbf{X}$ (cf. Jaffe, 1967). Both for simplicity, and because it would not alter the essential of our arguments, we are not doing so.

was Newton & Wigner's (1949) $\mathbf{X}_{\mathrm{NW, op}}$, which they used. However, it now seems clear that this is not so because $\mathbf{X}_{\mathrm{NW, op}}$ conflicts with the principles of relativity.† That is why a different $\phi'^{(-)}(\mathbf{X})$, corresponding to a more realistic $\mathbf{X}_{\mathrm{op}}$, should be used. However, in Kálnay (1970) we have shown that if (i) X^i_{op} exists and has the more essential required quantum mechanical properties and if (ii) the notion of localisation is consistent with the principles of relativity, then for the massive spin 1/2 particles the components X^i_{op} cannot commute with each other so that X^1, X^2, X^3 *do not belong to a complete set of observables*.‡ This last result, which we call (R), was also found in Kálnay (1971b) for the massless spin 1/2 and 1 particles. Additional support was found for (R) by the results of Kálnay & Torres (1971) and Kálnay & Torres (1973). Taking the result (R) as correct we can now point out that *a field* $\phi'^{(-)}(\mathbf{X})$, *such that equation* (1) *holds, cannot exist for the massive spin* 1/2 *particles nor for the massless spin* 1/2 *and* 1 *particles, i.e. for the more important 'elementary' particles with non-zero spin.*

Remarks: (a) It follows from Kálnay (1970, 1971b) that if $\mathbf{X}_{\mathrm{op}}$ exists and quantum mechanics still holds, but commutativity of the components of position is imposed for the non-zero spin systems under consideration, then the notion of localisation would preclude the physical equivalence of inertial frames of reference. (b) If position has a quantum mechanical meaning but equation (1) is rejected, then either the principles of quantum mechanics on which it is supported should be changed, or the use of the phrase 'field in configuration space' should be dropped altogether. If the notion of configuration has no meaning in microphysics, can it then achieve a meaning in a macroscopical assembly of microsystems? (See e.g. Kálnay, 1971a.) (c) It might be argued that equation (1) does not hold because of the virtual pair creation. However, as shown by Bunge (1970), 'quantum theories should be interpreted in such a way that they do not involve virtual processes or virtual quanta'. (d) A way out might be the discussion of localisation within a generalisation of quantum mechanics, as in the works of Broyles (1970, 1972) and Johnson (1969, 1971) (cf. Kálnay, 1971b). But then standard quantum mechanics must also be replaced by the new one when considering all other problems of microphysics. (e) Another way out is to reformulate field theory, for example, starting from the notion of 'extended type position' introduced in Kálnay & Toledo (1967), but excluding its 'limiting case' (Gallardo *et al.*, 1967).

References

Broyles, A. (1970). *Physical Review*, **D1**, 979.
Broyles, A. A. (1972). 'Space-Time Position Operators and Spin one-half Particles.' Preprint, Florida University.

† A review of the attempts to solve the localisation problem can be found in Kálnay (1971a). Some references on the related subjects of quantum field theory of localisation, local observables and localised fields can also be obtained there.

‡ We only consider particles without mass uncertainty.

Bunge, M. (1970). *International Journal of Theoretical Physics*, Vol. 3, No. 6, p. 507.
Gallardo, J. A., Kálnay, A. J., Stec, B. A. and Toledo, B. P. (1967). *Nuovo Cimento*, **49**, 393.
Jaffe, A. M. (1967). *Physical Review*, **158**, 1454.
Johnson, J. E. (1969). *Physical Review*, **181**, 1755.
Johnson, J. E. (1971). *Physical Review*, **D3**, 1735.
Kálnay, A. J. (1970). *Physical Review*, **D1**, 1092.
Kálnay, A. J. (1971a). The localization problem, in *Problems in the Foundations of Physics* (Ed. M. Bunge). Springer-Verlag, Berlin, Heidelberg, New York.
Kálnay, A. J. (1971b). *Physical Review*, **D3**, 2357.
Kálnay, A. J. and Toledo, B. P. (1967). *Nuovo Cimento*, **48**, 997.
Kálnay, A. J. and Torres, P. L. (1971). *Physical Review*, **D3**, 2977.
Kálnay, A. J. and Torres, P. L. (1973). *Physical Review*, **D7**, 1707.
Lurié, D. (1968). *Particles and Fields*. Interscience Publishers, New York, London, Sydney.
Newton, T. D. and Wigner, E. P. (1949). *Reviews of Modern Physics*, **21**, 400.
Wightman, A. S. and Schweber, S. S. (1955). *Physical Review*, **98**, 812.